

elements in LH also represent total labor costs, as indicated by the row and column labels shown below:

$$LH = \begin{bmatrix} \$70.50 & \$77 \\ \$43 & \$47 \end{bmatrix} \quad \begin{array}{l} \text{Labor costs per ski} \\ \text{CA} \quad \text{MD} \\ \text{Trick} \\ \text{Slalom} \end{array}$$



Figure 5 shows a solution to Example 9 on a spreadsheet.

	A	B	C	D	E	F
1		Labor-hours per ski			Hourly wages	
2		Assembly	Finishing		California	Maryland
3	Trick ski	5	1.5	Assembly	\$12	\$13
4	Slalom ski	3	1	Finishing	\$7	\$8
5		Labor costs per ski				
6		California	Maryland			
7	Trick ski	\$70.50	\$77.00			
8	Slalom ski	\$43.00	\$47.00			

Figure 5 Matrix multiplication in a spreadsheet: The command `MMULT(B3:C4, E3:F4)` produces the matrix in B7:C8

Matched Problem 10 Refer to Example 10. The company wants to know how many hours to schedule in each department in order to produce 2,000 trick skis and 1,000 slalom skis. These production requirements can be represented by either of the following matrices:

$$P = \begin{bmatrix} 2,000 & 1,000 \end{bmatrix} \quad Q = \begin{bmatrix} 2,000 \\ 1,000 \end{bmatrix} \quad \begin{array}{l} \text{Trick skis} \\ \text{Slalom skis} \end{array}$$

Using the labor-hour matrix L from Example 10, find PL or LQ , whichever has a meaningful interpretation for this problem, and label the rows and columns accordingly.

CONCEPTUAL INSIGHT

Example 10 and Matched Problem 10 illustrate an important point about matrix multiplication. Even if you are using a graphing calculator to perform the calculations in a matrix product, it is still necessary for you to know the definition of matrix multiplication so that you can interpret the results correctly.

Exercises 4.4

Skills Warm-up Exercises

W In Problems 1–14, perform the indicated operation, if possible. (If necessary, review the definitions at the beginning of Section 4.4.)

- $\begin{bmatrix} 1 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 10 \end{bmatrix}$
- $\begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 7 \\ -4 \end{bmatrix}$
- $\begin{bmatrix} 3 \\ 6 \end{bmatrix} + \begin{bmatrix} -1 & 9 \end{bmatrix}$
- $4 \begin{bmatrix} 2 & -6 & 1 \\ 8 & 5 & -3 \end{bmatrix}$
- $\begin{bmatrix} 2 & 0 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} 0 & -4 \\ -1 & 0 \end{bmatrix}$
- $\begin{bmatrix} -9 & 2 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 9 & 0 \\ 0 & 8 \end{bmatrix}$
- $7 \begin{bmatrix} 3 & -5 & 9 & 4 \end{bmatrix}$
- $\begin{bmatrix} 10 & 12 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix}$
- $\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$
- $\begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix}$

13. $\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} -2 & 5 \\ -1 & 3 \end{bmatrix}$

14. $\begin{bmatrix} -2 & 5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$

In Problems 15–22, find the matrix product. Note that each product can be found mentally, without the use of a calculator or pencil-and-paper calculations.

15. $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

16. $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$

17. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

18. $\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

19. $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$

20. $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

21. $\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

22. $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Find the products in Problems 23–30.

23. $\begin{bmatrix} 5 & -2 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} -3 \\ -4 \end{bmatrix}$

24. $\begin{bmatrix} -4 & 3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

25. $\begin{bmatrix} -3 \\ -4 \end{bmatrix} \begin{bmatrix} 5 & -2 \end{bmatrix}$

26. $\begin{bmatrix} -2 \\ 1 \end{bmatrix} \begin{bmatrix} -4 & 3 \end{bmatrix}$

27. $\begin{bmatrix} 3 & -2 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

28. $\begin{bmatrix} 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

29. $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} 3 & -2 & -4 \end{bmatrix}$

30. $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 \end{bmatrix}$

Problems 31–48 refer to the following matrices:

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -3 & 1 \\ -2 & 3 & 5 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$$

Perform the indicated operations, if possible.

31. AC

32. CA

33. AB

34. BA

35. B^2

36. C^2

37. $B + AD$

38. $C + DA$

39. $(0.1)DB$

40. $(0.2)CD$

41. $(3)BA + (4)AC$

42. $(2)DB + (5)CD$

43. $(-2)BA + (6)CD$

44. $(-1)AC + (3)DB$

45. ACD

46. CDA

47. DBA

48. BAD

49. If a and b are nonzero real numbers,

$$A = \begin{bmatrix} a & a \\ b & b \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} a & a \\ -a & -a \end{bmatrix}$$

find AB and BA .

50. If a and b are nonzero real numbers,

$$A = \begin{bmatrix} a & b \\ -a & -b \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} a & a \\ a & a \end{bmatrix}$$

find AB and BA .

51. If a and b are nonzero real numbers and

$$A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

find A^2 .

52. If a and b are nonzero real numbers and

$$A = \begin{bmatrix} ab & b - ab^2 \\ a & 1 - ab \end{bmatrix}$$

find A^2 .



In Problems 53 and 54, use a graphing calculator to calculate $B, B^2, B^3, \dots$ and $AB, AB^2, AB^3, \dots$. Describe any patterns you observe in each sequence of matrices.

53. $A = \begin{bmatrix} 0.3 & 0.7 \end{bmatrix}$ and $B = \begin{bmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix}$

54. $A = \begin{bmatrix} 0.4 & 0.6 \end{bmatrix}$ and $B = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$

55. Find a, b, c , and d so that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$$

56. Find w, x, y , and z so that

$$\begin{bmatrix} 4 & -2 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 0 & 5 \end{bmatrix}$$

57. Find a, b, c , and d so that

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

58. Find a, b, c , and d so that

$$\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 7 & -7 \end{bmatrix}$$

In Problems 59–62, determine whether the statement is true or false.

59. There exist two 1×1 matrices A and B such that $AB \neq BA$.

60. There exist two 2×2 matrices A and B such that $AB \neq BA$.

61. There exist two nonzero 2×2 matrices A and B such that AB is the 2×2 zero matrix.

62. There exist two nonzero 1×1 matrices A and B such that AB is the 1×1 zero matrix.



63. A square matrix is a **diagonal matrix** if all elements not on the principal diagonal are zero. So a 2×2 diagonal matrix has the form

$$A = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

where a and d are real numbers. Discuss the validity of each of the following statements. If the statement is always true, explain why. If not, give examples.

(A) If A and B are 2×2 diagonal matrices, then $A + B$ is a 2×2 diagonal matrix.

- (B) If A and B are 2×2 diagonal matrices, then $A + B = B + A$.
- (C) If A and B are 2×2 diagonal matrices, then AB is a 2×2 diagonal matrix.
- (D) If A and B are 2×2 diagonal matrices, then $AB = BA$.

 **64.** A square matrix is an **upper triangular matrix** if all elements below the principal diagonal are zero. So a 2×2 upper triangular matrix has the form

$$A = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

where a , b , and d are real numbers. Discuss the validity of each of the following statements. If the statement is always true, explain why. If not, give examples.

- (A) If A and B are 2×2 upper triangular matrices, then $A + B$ is a 2×2 upper triangular matrix.
- (B) If A and B are 2×2 upper triangular matrices, then $A + B = B + A$.
- (C) If A and B are 2×2 upper triangular matrices, then AB is a 2×2 upper triangular matrix.
- (D) If A and B are 2×2 upper triangular matrices, then $AB = BA$.

Applications

65. Cost analysis. A company with two different plants manufactures guitars and banjos. Its production costs for each instrument are given in the following matrices:

$$\begin{array}{cc} \text{Plant X} & \text{Plant Y} \\ \text{Guitar} & \text{Banjo} & \text{Guitar} & \text{Banjo} \\ \text{Materials} & \begin{bmatrix} \$47 & \$39 \end{bmatrix} & = A & \begin{bmatrix} \$56 & \$42 \end{bmatrix} & = B \\ \text{Labor} & \begin{bmatrix} \$90 & \$125 \end{bmatrix} & & \begin{bmatrix} \$84 & \$115 \end{bmatrix} & \end{array}$$

Find $\frac{1}{2}(A + B)$, the average cost of production for the two plants.

- 66. Cost analysis.** If both labor and materials at plant X in Problem 65 are increased by 20%, find $\frac{1}{2}(1.2A + B)$, the new average cost of production for the two plants.
- 67. Markup.** An import car dealer sells three models of a car. The retail prices and the current dealer invoice prices (costs) for the basic models and options indicated are given in the following two matrices (where “Air” means air-conditioning):

$$\begin{array}{cc} \text{Retail price} & \\ \text{Basic} & \text{AM/FM} & \text{Cruise} \\ \text{Car} & \text{radio} & \text{control} \\ \text{Model A} & \begin{bmatrix} \$35,075 & \$2,560 & \$1,070 & \$640 \end{bmatrix} \\ \text{Model B} & \begin{bmatrix} \$39,045 & \$1,840 & \$770 & \$460 \end{bmatrix} \\ \text{Model C} & \begin{bmatrix} \$45,535 & \$3,400 & \$1,415 & \$850 \end{bmatrix} \end{array} = M$$

$$\begin{array}{cc} \text{Dealer invoice price} & \\ \text{Basic} & \text{AM/FM} & \text{Cruise} \\ \text{Car} & \text{radio} & \text{control} \\ \text{Model A} & \begin{bmatrix} \$30,996 & \$2,050 & \$850 & \$510 \end{bmatrix} \\ \text{Model B} & \begin{bmatrix} \$34,857 & \$1,585 & \$660 & \$395 \end{bmatrix} \\ \text{Model C} & \begin{bmatrix} \$41,667 & \$2,890 & \$1,200 & \$725 \end{bmatrix} \end{array} = N$$

We define the markup matrix to be $M - N$ (**markup** is the difference between the retail price and the dealer invoice price). Suppose that the value of the dollar has had a sharp decline and the dealer invoice price is to have an across-the-board 15% increase next year. To stay competitive with domestic cars, the dealer increases the retail prices 10%. Calculate a markup matrix for next year's models and the options indicated. (Compute results to the nearest dollar.)

- 68. Markup.** Referring to Problem 67, what is the markup matrix resulting from a 20% increase in dealer invoice prices and an increase in retail prices of 15%? (Compute results to the nearest dollar.)
- 69. Labor costs.** A company with manufacturing plants located in Massachusetts (MA) and Virginia (VA) has labor-hour and wage requirements for the manufacture of three types of inflatable boats as given in the following two matrices:

$$\begin{array}{cc} \text{Labor-hours per boat} & \\ \text{Cutting} & \text{Assembly} & \text{Packaging} \\ \text{department} & \text{department} & \text{department} \\ M = \begin{bmatrix} 0.6 \text{ hr} & 0.6 \text{ hr} & 0.2 \text{ hr} \\ 1.0 \text{ hr} & 0.9 \text{ hr} & 0.3 \text{ hr} \\ 1.5 \text{ hr} & 1.2 \text{ hr} & 0.4 \text{ hr} \end{bmatrix} \begin{array}{l} \text{One-person boat} \\ \text{Two-person boat} \\ \text{Four-person boat} \end{array} \end{array}$$

$$\begin{array}{cc} \text{Hourly wages} & \\ \text{MA} & \text{VA} \\ N = \begin{bmatrix} \$17.30 & \$14.65 \\ \$12.22 & \$10.29 \\ \$10.63 & \$9.66 \end{bmatrix} \begin{array}{l} \text{Cutting department} \\ \text{Assembly department} \\ \text{Packaging department} \end{array} \end{array}$$

- (A) Find the labor costs for a one-person boat manufactured at the Massachusetts plant.
- (B) Find the labor costs for a four-person boat manufactured at the Virginia plant.
-  (C) Discuss possible interpretations of the elements in the matrix products MN and NM .
- (D) If either of the products MN or NM has a meaningful interpretation, find the product and label its rows and columns.

70. Inventory value. A personal computer retail company sells five different computer models through three stores. The inventory of each model on hand in each store is summarized in matrix M . Wholesale (W) and retail (R) values of each model computer are summarized in matrix N .

$$\begin{array}{cc} \text{Model} & \\ \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \\ M = \begin{bmatrix} 4 & 2 & 3 & 7 & 1 \\ 2 & 3 & 5 & 0 & 6 \\ 10 & 4 & 3 & 4 & 3 \end{bmatrix} \begin{array}{l} \text{Store 1} \\ \text{Store 2} \\ \text{Store 3} \end{array} \end{array}$$

$$\begin{array}{cc} \text{W} & \text{R} \\ N = \begin{bmatrix} \$700 & \$840 \\ \$1,400 & \$1,800 \\ \$1,800 & \$2,400 \\ \$2,700 & \$3,300 \\ \$3,500 & \$4,900 \end{bmatrix} \begin{array}{l} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \end{array}$$

- (A) What is the retail value of the inventory at store 2?
 (B) What is the wholesale value of the inventory at store 3?

-  (C) Discuss possible interpretations of the elements in the matrix products MN and NM .
 (D) If either product MN or NM has a meaningful interpretation, find the product and label its rows and columns.
 (E) Discuss methods of matrix multiplication that can be used to find the total inventory of each model on hand at all three stores. State the matrices that can be used and perform the necessary operations.
 (F) Discuss methods of matrix multiplication that can be used to find the total inventory of all five models at each store. State the matrices that can be used and perform the necessary operations.

- 71. Cereal.** A nutritionist for a cereal company blends two cereals in three different mixes. The amounts of protein, carbohydrate, and fat (in grams per ounce) in each cereal are given by matrix M . The amounts of each cereal used in the three mixes are given by matrix N .

$$M = \begin{array}{cc|l} \text{Cereal A} & \text{Cereal B} & \\ \hline 4 \text{ g/oz} & 2 \text{ g/oz} & \text{Protein} \\ 20 \text{ g/oz} & 16 \text{ g/oz} & \text{Carbohydrate} \\ 3 \text{ g/oz} & 1 \text{ g/oz} & \text{Fat} \end{array}$$

$$N = \begin{array}{ccc|l} \text{Mix X} & \text{Mix Y} & \text{Mix Z} & \\ \hline 15 \text{ oz} & 10 \text{ oz} & 5 \text{ oz} & \text{Cereal A} \\ 5 \text{ oz} & 10 \text{ oz} & 15 \text{ oz} & \text{Cereal B} \end{array}$$

- (A) Find the amount of protein in mix X.
 (B) Find the amount of fat in mix Z.
 (C) Discuss possible interpretations of the elements in the matrix products MN and NM .
 (D) If either of the products MN or NM has a meaningful interpretation, find the product and label its rows and columns.

-  **72. Heredity.** Gregor Mendel (1822–1884) made discoveries that revolutionized the science of genetics. In one experiment, he crossed dihybrid yellow round peas (yellow and round are dominant characteristics; the peas also contained genes for the recessive characteristics green and wrinkled) and obtained peas of the types indicated in the matrix:

$$\begin{array}{cc|l} & \text{Round} & \text{Wrinkled} & \\ \hline \text{Yellow} & 315 & 101 & \\ \text{Green} & 108 & 32 & \end{array} = M$$

Suppose he carried out a second experiment of the same type and obtained peas of the types indicated in this matrix:

$$\begin{array}{cc|l} & \text{Round} & \text{Wrinkled} & \\ \hline \text{Yellow} & 370 & 128 & \\ \text{Green} & 110 & 36 & \end{array} = N$$

If the results of the two experiments are combined, discuss matrix multiplication methods that can be used to find the following quantities. State the matrices that can be used and perform the necessary operations.

- (A) The total number of peas in each category
 (B) The total number of peas in all four categories
 (C) The percentage of peas in each category

- 73. Politics.** In a local California election, a public relations firm promoted its candidate in three ways: telephone calls, house calls, and letters. The cost per contact is given in matrix M , and the number of contacts of each type made in two adjacent cities is given in matrix N .

$$M = \begin{array}{c|l} \text{Cost per contact} & \\ \hline \$1.20 & \text{Telephone call} \\ \$3.00 & \text{House call} \\ \$1.45 & \text{Letter} \end{array}$$

$$N = \begin{array}{ccc|l} \text{Telephone call} & \text{House call} & \text{Letter} & \\ \hline 1,000 & 500 & 5,000 & \text{Berkeley} \\ 2,000 & 800 & 8,000 & \text{Oakland} \end{array}$$

- (A) Find the total amount spent in Berkeley.
 (B) Find the total amount spent in Oakland.
 (C) Discuss possible interpretations of the elements in the matrix products MN and NM .
 (D) If either product MN or NM has a meaningful interpretation, find the product and label its rows and columns.
 (E) Discuss methods of matrix multiplication that can be used to find the total number of telephone calls, house calls, and letters. State the matrices that can be used and perform the necessary operations.
 (F) Discuss methods of matrix multiplication that can be used to find the total number of contacts in Berkeley and in Oakland. State the matrices that can be used and perform the necessary operations.
-  **74. Test averages.** A teacher has given four tests to a class of five students and stored the results in the following matrix:

$$\begin{array}{c|cccc} & \text{Tests} & & & & \\ & 1 & 2 & 3 & 4 & \\ \hline \text{Ann} & 78 & 84 & 81 & 86 & \\ \text{Bob} & 91 & 65 & 84 & 92 & \\ \text{Carol} & 95 & 90 & 92 & 91 & \\ \text{Dan} & 75 & 82 & 87 & 91 & \\ \text{Eric} & 83 & 88 & 81 & 76 & \end{array} = M$$

Discuss methods of matrix multiplication that the teacher can use to obtain the information indicated below. In each case, state the matrices to be used and then perform the necessary operations.

- (A) The average on all four tests for each student, assuming that all four tests are given equal weight
 (B) The average on all four tests for each student, assuming that the first three tests are given equal weight and the fourth is given twice this weight
 (C) The class average on each of the four tests

Answers to Matched Problems

1. $\begin{bmatrix} 1 & 5 \\ 0 & -2 \\ 2 & 1 \end{bmatrix}$

3. $a = -6$
 $b = 7$
 $c = 9$
 $d = -1$

5. (A) $\begin{bmatrix} \$235,000 & \$422,000 \\ \$372,000 & \$298,000 \end{bmatrix}$

(B) $\begin{bmatrix} \$145,000 & \$268,000 \\ \$160,000 & \$254,000 \end{bmatrix}$

(C) $\begin{bmatrix} \$9,500 & \$17,250 \\ \$13,300 & \$13,800 \end{bmatrix}$

2. $[-1 \quad -1 \quad 4]$

4. $\begin{bmatrix} 13 \\ 2 \\ 35 \end{bmatrix}$

6. $[8]$

7. $[5 \quad 1.5] \begin{bmatrix} 9 \\ 6 \end{bmatrix} = [54]$, or \$54

8. (A) Not defined

(B) $\begin{bmatrix} 2 & 2 & -1 & 2 \\ 1 & 6 & 12 & -4 \\ -1 & 0 & 3 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} -6 & -12 \\ 3 & 6 \end{bmatrix}$ (E) $[11]$

(F) $\begin{bmatrix} 12 & -8 & 4 \\ 6 & -4 & 2 \\ 9 & -6 & 3 \end{bmatrix}$

9. $a = 4, c = 8, b = 9, d = -2$

Assembly Finishing

10. $PL = [13,000 \quad 4,000]$ Labor-hours

4.5 Inverse of a Square Matrix

- Identity Matrix for Multiplication
- Inverse of a Square Matrix
- Application: Cryptography

Identity Matrix for Multiplication

Does the set of all matrices of a given size have an identity element for multiplication? That is, if M is an arbitrary $m \times n$ matrix, does there exist an identity element I such that $IM = MI = M$? The answer, in general, is no. However, the set of all **square matrices of order n** (matrices with n rows and n columns) does have an identity element.

DEFINITION Identity Matrix

The **identity element for multiplication** for the set of all square matrices of order n is the square matrix of order n , denoted by I , with 1's along the principal diagonal (from the upper left corner to the lower right) and 0's elsewhere.

For example,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

are the identity matrices for all square matrices of order 2 and 3, respectively.



Most graphing calculators have a built-in command for generating the identity matrix of a given order (see Fig. 1).

```
identity 2
  [[1 0]
   [0 1]]
identity 3
  [[1 0 0]
   [0 1 0]
   [0 0 1]]
```

Figure 1 Identity matrices